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The Floquet theory of secondary instability in shear flows has been further developed and applied to a variety of flows. The linear theory has been extended to explain and quantitatively analyze the observed combination resonance in boundary layers. Numerical methods for the study of secondary instability in unbounded flows have been developed and applied to the viscous and inviscid mixing layer. The linear theory has been formulated for a variety of spatially periodic flows that include Görtler vortices and oblique waves. Applications await accounting for nonparallel effects. A new approach to analyzing nonparallel flows based on parabolic partial differential equations has been successfully applied to the primary stability problem. A perturbation method has been developed to reveal the nonlinear interactions that lead to breakdown of the laminar flow. This method permits prediction of the transition location in a given disturbance environment.

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Air Force Office of Scientific Research
Contract F49620-87-K-0005

Three-Dimensional Structure of Boundary Layers in Transition to Turbulence

by

Thorwald Herbert

Department of Mechanical Engineering and
Department of Aeronautical & Astronautical Engineering
The Ohio State University
Columbus, Ohio 43210, U.S.A.

February 1989

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Summary

The Floquet theory of secondary instability in shear flows has been further developed and applied to a variety of flows. The linear theory has been extended to explain and quantitatively analyze the observed combination resonance in boundary layers. Numerical methods for the study of secondary instability in unbounded flows have been developed and applied to the viscous and inviscid mixing layer. The linear theory has been formulated for a variety of spatially periodic flows that include Görtler vortices and oblique waves. Applications await accounting for nonparallel effects. A new approach to analyzing nonparallel flows based on parabolic partial differential equations has been successfully applied to the primary stability problem. A perturbation method has been developed to reveal the nonlinear interactions that lead to breakdown of the laminar flow. This method permits prediction of the transition location in a given disturbance environment.

1. Research Objectives

The project "Three-Dimensional Structure of Boundary Layers in Transition to Turbulence" under AFOSR Contract F49620-87-K-0005 was originally planned as a three-year effort with the working period 03/01/87 - 02/28/90 and Thorwald Herbert as principal investigator. Owing to Th. Herbert's accepting a new position at The Ohio State University (OSU) beginning with the academic year 1987/88 on October 1, 1987, only a part of the program was conducted at the Virginia Polytechnic Institute and State University (VPI). The contract was terminated by 02/29/88 with a no-cost extension of the working period until 06/30/88. The work at VPI was supported by the first-year budget amount of the original proposal. Roger Simpson acted as principal investigator at VPI after Th. Herbert joined OSU. After split and rearrangement of the research plan, years two and three were re-initiated as grant AFOSR-88-0186 at OSU with Th. Herbert as principal investigator.

A detailed description of the research objectives has been given in section 3.3 of the original proposal. Overall, the work aimed at gaining insight into the intricate process of laminar-turbulent transition in plane shear flows, especially in boundary layers. Our research goals during the three-year period of the contract were comprised of the following three topics:

(1) Development and application of the Floquet theory for spanwise periodic flows as they occur in the presence of counter-rotating longitudinal vortices (e. g. Görtler vortices) and for flows in the presence of oblique waves. These studies extend the catalogue of mechanisms relevant to the onset of transition, shed light on the breakdown of longitudinal vortices in the viscous sublayer of turbulent flows, and provide the basis for analyzing the evolution of wave packets into turbulent spots.

(2) Incorporation of the temporal or spatial growth of primary disturbances into the analysis. This extension of the theory improves the quantitative prediction of disturbance characteristics especially for boundary layers with adverse pressure gradients where primary disturbances exhibit rapid growth.

(3) Analysis of nonlinear interactions between primary and secondary disturbances by using innovative perturbation techniques and numerical (spectral) methods. This work provides a description of the flow field in the nonlinear three-dimensional stage of transition, threshold conditions for self-sustained three-dimensional growth and insight into the feedback loops of the transition process. In addition, the combined use of numerical and perturbation analysis reveals the origin of suppression and selection amongst competing modes.

2. Research Achievements

After initiation of this contract, work was simultaneously conducted on all three topics. Rapid progress was made until late summer 1987 when our team was divided, with only G. Santos and J. Crouch remaining at VPI. The work was divided such that optimum profit to the research program could be combined with successful completion of the Ph.D. theses at VPI within the extended working period of the contract.

The Floquet theory of secondary instability in shear flows has been previously applied only to mean flows with superposed finite-amplitude TS waves, e.g. with the periodic direction in the mean-flow direction. We have formulated the theory for flows that are:

(a) periodic in the spanwise direction (in the presence of Görtler vortices), where the wave vector is normal to the mean-flow direction;

(b) periodic in an oblique direction (in the presence of an oblique wave), where the wave vector includes an angle ψ to the mean-flow direction;

(c) doubly periodic (in the presence of two oblique waves under angles $\pm\psi$), where the resultant wave vector is in mean-flow direction.

Various formal conclusions have been drawn with some of them awaiting numerical verification. The arrangement (c) appears drastically different from all other cases since (under the parallel-flow assumption) the vortex arrays created by the primary disturbances are no longer invariant under a translation along the wave crests. The question arises whether this property favors the amplification of known secondary modes with preferred (instead of broad-band) spanwise wave lengths or whether the development of the three-dimensional vorticity field follows a new route to transition. The numerical studies to answer this question were not completed at VPI. (These studies are now performed at OSU).

The second major formal conclusion concerns the effect of nonparallelism of boundary-layer and other shear flows on the secondary instability. For the case of TS waves, the nonparallel effect is relatively weak in comparison to the convective terms owing to the streamwise oscillation. This ratio changes as the streamwise wavenumber decreases to the extend that the normal-mode approach is inapplicable for Görtler vortices (Hall 1983). Further, for TS waves, the secondary instability originates from combined tilting and stretching of the vortex array in the mean-shear flow. As the wave angle increases, this mechanism changes since, ultimately, the vortices are aligned with the mean-flow direction. The secondary growth rates, therefore, decrease with increasing wave angle, and the assumption, that secondary modes grow fast in comparison with primary modes, is no longer satisfied. Consequently, we need not only to consider the nonparallelism of the mean flow but also the effect of primary growth on the secondary instability.

The existing nonparallel stability theories (Bouthier 1973, Gaster 1974, Saric & Nayfeh 1977) are not only controversial but conceptually insufficient to overcome the existing problems. Hall (1983) has suggested the analysis of Görtler vortices based on parabolic differential equations for spanwise periodic boundary layers along a curved wall. This concept cannot be applied to streamwise periodic primary disturbances which are governed by the elliptic Navier-Stokes equations. We have developed a new approach that locally decomposes the solution into a multiplicative wave component and a streamwise slowly varying component that is governed by the boundary-layer equations. We obtain a system of parabolic differential equations for the stability problem. This system is valid over a limited streamwise domain which is sufficiently large to analyze situations with strong streamwise disturbance growth with a marching scheme. First results on the linear nonparallel stability of the Blasius flow were presented at the 1987 APS meeting (Herbert & Bertolotti 1987). Formal and numerical results shed new light on the nonparallel effects as well as on previous work. Besides accounting for nonparallelism, the method can optionally account for nonlinear effects - for the first time in the history of stability theory. The detailed evaluation of previous work, comparison with experiments (Schubauer & Skramstad 1943, Wortmann 1955, Ross et al. 1970, Kachanov et al. 1977) and numerical results (Fasel 1976, Fasel, Rist & Konzelmann 1987), and the separation of nonparallel and nonlinear effects have been partially conducted at OSU and a paper is near completion (Bertolotti & Herbert 1989). A coordinated study of this problem is performed by P. Spalart at NASA Ames Research Center using a spectral Navier-Stokes code. The planned back-to-back publication of our and Spalart's results will settle the controversial issues of previous work. While Spalart's computations are very expensive, our marching scheme is far more efficient than the traditional parallel-flow based integration of Orr-Sommerfeld solutions. This achievement is considered of major importance for the analysis of compressible boundary-layer stability.

The linear theory has been extended to explain and quantitatively analyze the observed combination resonance in boundary layers. Leaning on the weakly nonlinear stability theory, combination resonance is usually considered a nonlinear phenomenon. In the framework of the Floquet theory, however, it turns out that combination resonance is a linear phenomenon which results from the requirement of a real solution for the physical system. Detuned modes which are shifted by Δ with respect to the subharmonic frequency (or wavenumber) are associated with complex eigenvalues and therefore appear in complex conjugate pairs. The complex conjugate solution is shifted by $-\Delta$. For relevant amplitudes, moderately detuned modes grow with rates only slightly less than the subharmonic growth rate. With a uniform disturbance background, combination resonance therefore leads to a broad spectral peak centered at the subharmonic frequency as found by Kachanov & Levchenko (1984) and Saric (personal communication).

The results on combination resonance together with the broad-band nature of the secondary instability mechanism with respect to spanwise wave lengths are important for our understanding of transition. While earlier hypotheses implied selective resonances and hence specific "most dangerous" background disturbances, our results show that the secondary instability mechanism is not strongly selective, neither with respect to the spanwise nor to the streamwise scale. In other words, the secondary instability mechanism amplifies most every disturbance except for those with very large and very small spanwise wave lengths. We also conclude that the scale of the TS wave is not critical for the appearance of secondary instability in a given environment.

Besides the phenomena in the boundary layer, we have studied the catalogue of secondary modes in the mixing layer both in the inviscid limit and viscous cases. This work served to verify existing results of Pierrehumbert & Widnall (1982), to evaluate the effect of different basic flows (Stuart vortices versus finite-amplitude Orr-Sommerfeld modes), to adapt our numerical methods to infinite flow domains, and to investigate the effect of strong amplification of the primary mode on the secondary growth rates. The essence of Pierrehumbert & Widnall's results for Stuart vortices was confirmed although the accuracy of their inviscid analysis suffers at small growth rates from interference with the continuous spectrum. While this accuracy can be improved by higher spectral approximations, a more elegant (and less costly) way is the introduction of an extremely small viscosity that does not affect the discrete modes but moves the continuous spectrum into the stable domain.

Existing work on secondary instability of the mixing layer is for neutral (thus relatively uninteresting) conditions. Various studies were performed to incorporate the rapid growth of the primary wave into the analysis. These attempts suggested the use of partial differential equations to describe the problem. While various approaches led to the same results, inconsistencies with the numerical work of Metcalfe et al. (1987) could not be consolidated. Meanwhile, our results have been verified by S. Ragab (personal communication).

One of the major goals to be pursued within this contract was the verification of a positive feedback loop earlier suggested by analysis of the energy transfer between mean flow, two-dimensional wave, and three-dimensional disturbances (Croswell 1985). Since secondary instability ceases with the TS wave, criteria for the onset of breakdown must involve a nonlinear interaction between secondary and primary disturbances. More specifica'.y, this interaction must either modify the mean flow such that the primary mode does not decay, or it must produce a two-dimensional field similar to that of the TS wave suitable to maintain the growth of the secondary mode.

Since attempts failed to describe the secondary instability with weakly nonlinear methods, we choose perturbation expansions in terms of two-dimensional modes and a three-dimensional secondary mode. Such expansions bring about the dilemma that the amplitude of the two-dimensional mode should be constant (to determine the secondary mode) as well as variable (to capture the development of the two-dimensional field). This dilemma was resolved by a pseudo-marching procedure that is described in detail by Crouch (1988). For given initial amplitudes of two- and three-dimensional disturbances, the method reproduces the development of the disturbance field up to amplitudes of 6% to 8% where experiments show the onset of spikes - the operational definition of breakdown. The theoretical predictions are consistent with the experiments of Kachanov et al (1977), Kachanov & Levchenko (1984), Corke & Mangano (1987), and Cornelius (1985) as well as with the computer simulations of transition by Spalart & Yang (1986).

The essence of the mechanism of self-sustained three-dimensional growth is revealed at the lowest, second-order interaction. The deformation the mean flow prevents the decay of the TS wave as the three-dimensional disturbances gain sufficient strength. Instead of the rapid growth observed in the experiments, however, the TS wave amplitude maintains an almost constant level. The observed growth is due to an additional two-dimensional component forced by the self-interaction of the secondary modes. The level of the TS amplitude and hence the growth rate of the three-dimensional mode depend on the amplitudes, Reynolds number, and frequency at the onset of the nonlinear interaction. Details of the mechanism and the subtle differences for subharmonic and peak-valley splitting modes of secondary instability are discussed by Crouch (1988). The material is currently prepared for publication.

The perturbation analysis is capable of reliably predicting whether self-sustained growth develops and at which streamwise position, provided the initial amplitudes are known. The prediction of the streamwise position for the rapid simultaneous growth of all disturbance components is equivalent to predicting breakdown or the transition Reynolds number within a negligible error of one or two TS wavelengths. We consider this capability and the insight into the late stage of nonlinear instability one of the major achievements under this contract.

3. Personnel

During the working period, the following personnel were partly supported under Contract F49620-87-K-0005:

Thorwald Herbert, Professor, Principal Investigator
German Santos, Graduate Student (Ph.D. level)
Jeffrey Crouch, Graduate Student (Ph.D. level)
Fabio Bertolotti, Graduate Student (Ph.D. level)
Charlotte R. Hawley, Research Specialist

German Santos developed the concept of combination resonance for boundary layers and studied the application to plane shear flows. He developed the numerical tools to study secondary instability in parallel flows in an infinite domain (mixing layer, wake). G. Santos investigated the role of the parallel-flow assumption in the secondary stability analysis. He received his Ph.D. degree in November 1987 at VPI and returned to his home country, Columbia.

Jeff Crouch developed formulation and computer programs for an innovative perturbation analysis of nonlinear secondary instability. The analysis yields threshold conditions for the onset of self-sustained growth of three-dimensional disturbances and quantitatively describes the transitional flow up to the breakdown stage. J. Crouch received his Ph.D. degree in April 1988 at VPI and is now a Postdoctoral Associate at the Naval Research Laboratory.

Fabio Bertolotti developed the software system for computer animation of unstable boundary-layer flows on Apollo work stations and evaluated concepts for the analysis of spatial growth in nonparallel flows. Since October 1987, he has been a Graduate Research Assistant at OSU and continues his cooperation in the research program.

Charlotte Hawley was responsible for computer operations and software, project administration, and technical manuscripts. Since December 1987, she has been a Research Assistant at OSU and continues her cooperation in the research program.

4. Publications

The following publications, reports, and communications were prepared with support by contract F49620-87-K-0005:

(1) "On the Mechanisms of Transition in Boundary Layers," by Th. Herbert and G. R. Santos, AIAA Paper No. 87-1201 (1987).

(2) "Stability Analysis of Nonparallel Boundary Layers," by Th. Herbert and F. P. Bertolotti, Bull. Amer. Phys. Soc., Vol. 32, p. 2079 (1987).

(3) "Nonlinear Wave Interactions in Fluids," Editors: R. W. Miksad, T. R. Akylas, and Th. Herbert, ASME, AMD-Vol. 87 (1987).

(4) "Instability Mechanisms in Shear Flow Transition," by B. J. Bayly, S. A. Orszag, and Th Herbert, Ann. Rev. Fluid Mech., Vol. 20, pp 359-391 (1988).

(5) "Secondary Instability of Boundary Layers," by Th. Herbert, Ann. Rev. Fluid Mech., Vol 20, pp. 487-526 (1988).

(6) "Onset of Transition in Boundary Layers," by Th. Herbert, Int. J. Num. Meth. Fluids, Vol. 8, pp. 1151-1164 (1988).

(7) "Symbolic Computations with Spectral Methods," by Th. Herbert, in: Symbolic Computation in Fluid Mechanics and Heat Transfer, ASME, AMD-Vol. 97, pp. 25-32 (1988).

(8) "Nonlinear Evolution of Secondary Instabilities in Boundary Layers," by J. D. Crouch and Th. Herbert, Bull. Amer. Phys. Soc., Vol. 33, p. 2260 (1988).

(9) "Exploring Transition by Computer," by Th. Herbert, J. Appl. Num. Math., (1988). To appear.

(10) "Studies of Transition in Boundary Layers," by Th. Herbert and R. J. Bodonyi, AIAA Paper No. 89-0034 (1989).

The following papers reporting results obtained under the support by this contract are in preparation:

(11) "Combination Resonance in Unstable Boundary Layers," by G. Santos and Th. Herbert, to be submitted to J. Fluid Mech.

(12) "Temporal and Spatial Growth of Secondary Disturbances," by F. P. Bertolotti and Th. Herbert, to be submitted to Phys. Fluids.

(19) "Weakly Nonlinear Analysis of Secondary Instability in the Blasius Boundary Layer," by J. D. Crouch and Th. Herbert, to be submitted to J. Fluid Mech.

(16) "A Study on Visualizations of Boundary-Layer Transition," by F. P. Bertolotti and Th. Herbert, to be submitted to Exp. Fluids.

5. Degrees Awarded

The following degrees were awarded for research work conducted under this contract:

(1) German Santos, "Studies on Secondary Instability in Shear Flows," Ph.D. Thesis, Major Professor: Th. Herbert, Virginia Polytechnic Institute and State University, Blacksburg, Virginia, November 1987.

(2) J. D. Crouch, "The Nonlinear Evolution of Secondary Instabilities in Boundary Layers," Ph.D. Thesis, Major Professor: Th. Herbert, Virginia Polytechnic Institute and State University, Blacksburg, Virginia (April 1988).

6. Technical Presentations

The following papers were presented at meetings, conferences and seminars:

(1) "On the Mechanism of Transition in Boundary Layers," by Th. Herbert and G. R. Santos, AIAA 19th Fluid Dynamics, Plasma Dynamics and Laser Conference, Honolulu, Hawaii (June 1987).

(2) "Onset of Transition in Boundary Layers," by Th. Herbert, Invited Lecture, Fifth International Conference on Numerical Methods in Laminar and Turbulent Flow, Montreal, Canada (July 1987).

(3) "Stability Analysis of Nonparallel Boundary Layers," by Th. Herbert and F. P. Bertolotti, 40th Annual Meeting of the Division of Fluid Dynamics, American Physical Society, Eugene, Oregon (Nov. 1987).

(4) "Weakly Nonlinear Theory and Floquet Theory of Secondary Instability in Shear Flows," by Th. Herbert, Symposium on Nonlinear Wave Interactions in Fluids, ASME Winter Annual Meeting, Boston, Massachusetts (Dec. 1987).

(5) Early Stages of Transition in Boundary Layers," by Th. Herbert, Department of Aerospace Engineering, Texas A&M Univ., College Station, TX (Feb. 1988).

(6) "Three-Dimensional Structure of Boundary Layer Transition", by Th. Herbert, AFOSR Meeting on Turbulence Research, Los Angeles, CA (June 1988).

(7) "Exploring Transition by Computer," by Th. Herbert, Invited Lecture, SAE, Aerotech '88, Anaheim, CA (Oct. 1988).

(8) "Nonlinear Evolution of Secondary Instabilities in Boundary Layers," by J. D. Crouch and Th. Herbert, 41st Annual Meeting of the Division of Fluid Dynamics, American Physical Society, Buffalo, NY (Nov. 1988).

(9) "Studies of Transition in Boundary Layers," by Th. Herbert and R. J. Bodonyi, Invited Lecture, AIAA 27th Aerospace Sciences Meeting, Reno, Nevada (Jan. 1989).

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